Optical bistability induced by mirror absorption: measurement of absorption coefficients at the sub-ppm level

K. An, B. A. Sones, C. Fang-Yen, R. R. Dasari, and M. S. Feld

George R. Harrison Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received May 2, 1997

We have observed optical bistability caused by absorption-induced thermal expansion of mirrors forming a Fabry–Perot interferometer. From the resulting anomalous transmission line shapes, absorption coefficients of the mirror coatings as low as 0.2 parts in 10^6 (ppm) have been successfully measured. © 1997 Optical Society of America

Optical bistability is a well-known phenomenon in which absorption or dispersion in a medium in a resonator induces an optical path-length change that is proportional to the intracavity light intensity in such a way that cavity transmission exhibits more than one stable operating point.[1,2] Hence, optical nonlinearity of the medium is essential. However, purely classical optical bistability exists that does not require the medium to be nonlinear. One such mechanism is based on the radiation pressure exerted by the intracavity field on a movable mirror of the cavity.[3,4] There have been numerous studies of thermally induced optical bistability,[2] particularly in a thin sample with appreciable absorption in a Fabry–Perot interferometer. The thermal expansion of the sample, which is proportional to the intracavity intensity, changes the optical path length. Thin samples such as etalons and optical filters can serve as a cavity as well as an absorbing medium. There is, however, no fundamental reason why a medium is needed at all, since any Fabry–Perot interferometer has its own absorption, however small. Thus, under proper conditions it should be possible to observe optical bistability induced by thermal expansion of the mirrors themselves.

Light absorption into a mirror coating can generate heat that diffuses into the mirror substrate and causes thermal expansion of the mirrors. If the two mirrors that form the resonator are mounted at their outer uncoated ends, such expansion changes the cavity length, which is the distance from one coated surface to the other [Fig. 1(a)]. Although plausible, optical bistability owing to mirror absorption has not been reported, to our knowledge. To detect it, the absorption has to be highly localized, so that a small volume is heated and cooled fairly quickly (as in the above-mentioned thin-filter–etalon experiments). The cavity should also possess a narrow linewidth, i.e., high finesse, to resolve a small frequency shift that is due to the thermal expansion (of the order of 0.1 nm) of such a small volume within the mirrors. A supercavity, which exhibits an ultrahigh Q ($\sim 10^9$), can meet these requirements.

The importance of mirrors with ultralow loss in science and technology has grown recently. These mirrors are used in laser gyroscopes, gravitational-wave detectors, and cavity QED experiments.[5] Further applications are expected. There exist demands for ultrahigh optical Q, equivalent to ultrahigh finesse. Characterizing supercavity mirrors is essential for advancing this technology. Absorption and scatter, the loss mechanisms in these mirrors, are small, being at the parts-in-10^6 (ppm) level. However, these mechanisms are not well understood or controlled. Detection of the absorption coefficient has been based mainly on various thermal-refraction processes.[6] Absorption of as little as 1 ppm can be measured with extreme care and careful calibration against a known standard.[7] However, accurate measurement of mirror absorption at levels below 1 ppm has not been possible.

In this Letter we report on the optical bistability caused by light absorption into mirror coatings. Furthermore, based on this effect, we have developed a technique to measure the absorption coefficients of coatings of the supercavity.

Consider a circular, highly reflective mirror of radius R and thickness L, with both dimensions much larger than the wavelength of light, λ. The mirror has multiple coating layers, the number of which can vary from one to several tens. The thickness of the coating layers is much smaller than R and L. Assume that an intense laser beam is incident

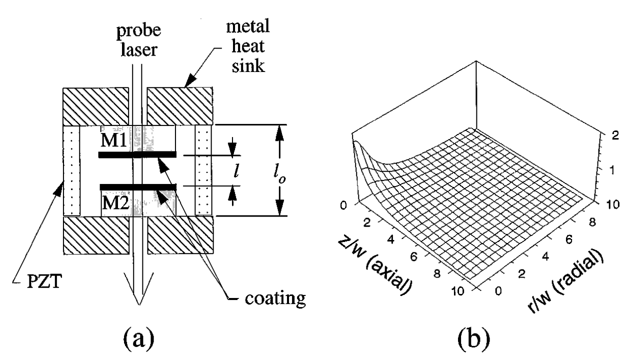

Fig. 1. (a) Cavity configuration. PZT, piezoelectric transducer; M1, M2 mirrors. (b) Temperature distribution in the mirror substrate (in units of $P_i A/2\pi w\kappa$) in cylindrical coordinates, with $w/L = 0.0075$.

0146-9592/97/181433-03$10.00/0

at the center of the mirror. The laser beam has an intensity mode waist $w(\lambda \ll w \ll L, R)$. The laser beam intensity decreases exponentially as the beam penetrates the coating layers. As a result, light absorption occurs only in the coating, and the laser intensity in the substrate is negligible. The laser power that is absorbed is $P_i A$, with P_i being the laser beam power and A the absorption coefficient of the coating. The absorbed energy is converted into heat, which propagates from the region of absorption to the rest of the mirror according to the heat-diffusion equation

$$\frac{\partial \psi}{\partial t} = \frac{\kappa}{s} \nabla^2 \psi \,, \tag{1}$$

with ψ being the temperature, κ the thermal conductivity, and s the specific heat (per unit volume) of the mirror substrate. We assume that the heat is dissipated only through the bottom of the mirror into a heat reservoir, the temperature of which is fixed at ψ_0. Figure 1(b) depicts temperature distribution in the mirror, which we obtained analytically by solving Eq. (1) in cylindrical coordinates.

For a mirror with a finite length L but with $R/w \gg 1$, the length change that is due to thermal expansion along the mirror axis is

$$\Delta L = C_{\mathrm{ex}} \int_0^L [\psi(\rho = 0, z) - \psi_0] \mathrm{d}z \simeq \frac{C_{\mathrm{ex}} C_0 P_i \mathcal{A}}{\pi \kappa} \,, \tag{2}$$

with C_{ex} being the thermal-expansion coefficient of the mirror substrate and C_0 a constant that is of the order of unity, depending only on the geometry of the mirror:

$$C_0 = \frac{1}{2} \int_0^\infty \mathrm{d}q \, \exp(-\epsilon^2 q^2/4) \frac{\cosh q - 1}{q \cosh q} \,, \tag{3}$$

with $\epsilon \equiv w/L$. Since the thickness of the coating layers is much smaller than that of the substrate and w, we can safely neglect the thermal expansion of the coating.

Suppose that two such mirrors are put together facing each other in a Fabry–Perot configuration. Assume that the distance from the outer end of one mirror to the outer end of the other mirror, l_0, is held fixed (although later this distance will be scanned). Thermal expansion of the mirrors can then lead to a change in the mirror separation l [Fig. 1(a)]. Consider a probe laser beam with power P_o incident at the center of the mirror of the cavity. The power P_i of each traveling-wave component of the standing wave inside the cavity is related to the incident power, P_o, by

$$P_i = P_o \frac{\mathcal{T}}{(1 - \mathcal{R})^2} \mathcal{L}(\omega) \,, \tag{4}$$

where $\mathcal{R}$ and $\mathcal{T}$ are the reflectance and the transmittance of the mirrors, respectively, and $\mathcal{L}(\omega)$ is the cavity-transmission line shape modified by the thermal expansion, with ω being the frequency of the laser beam. Without the thermal expansion this line shape is simply a Lorentzian. However, the thermal expansion modifies the boundary condition for the field: If the two mirrors are identical, they will expand toward each other equally by an amount ΔL, which is given by Eq. (2). This change of the cavity length leads to a shift in the cavity resonance frequency by

$$\Delta \omega_c = 2\omega_c \Delta L/l = \left[\frac{2\omega_c C_{\mathrm{ex}} C_0 \mathcal{T} \mathcal{A} P_0}{\pi \kappa l (1 - \mathcal{R})^2} \right] \mathcal{L}(\omega) \,. \tag{5}$$

This shift, which is proportional to the intracavity intensity, is the source of the optical bistability. Defining $y(x)$ as $\mathcal{L}(\omega) \equiv y(x)|_{x=(\omega-\omega_c)/\gamma_c}$, with cavity-detuning parameter x, where ω_c and γ_c are the cavity resonance frequency and linewidth (half-width in radians per second), respectively, we obtain a self-consistency relation for the resulting line-shape function $y(x)$:

$$y(x) = \frac{1}{1 + (x - \beta y)^2} \,, \tag{6}$$

where

$$\beta = \frac{8 C_{\mathrm{ex}} C_0 \mathcal{T} \mathcal{A} P_o \mathcal{F}^3}{\pi^3 \lambda \kappa} \,, \tag{7}$$

with $\mathcal{F} = \pi/(1 - \mathcal{R})$ the finesse of the cavity. In Eq. (6) βy accounts for the intensity-dependent frequency shift, which is given by Eq. (5). We can obtain the line-shape function $y(x)$ graphically by tilting the Lorentzian $y_0(x) = 1/(1 + x^2)$, as shown in Fig. 2. For a given cavity-laser detuning, two stable cavity-transmission states can exist in the steady state, depending on cavity-scan history. If the cavity is scanned slowly (adiabatic limit) in the direction of increasing l_0, the cavity transmission increases to a peak value $y = 1$, which occurs approximately at $x = \beta$ and then abruptly drops to nearly zero [path (i) in Fig. 2]. This peak value of cavity detuning, $\beta \gamma_c$ (in units of frequency) is hereafter called the cavity frequency shift. On the other hand, if the cavity is scanned in the direction of decreasing cavity length, the observed linewidth is much narrower, and the peak value is much smaller than unity [path (ii) in Fig. 2]. The hysteresis curve, plotted in the usual form of output versus power input, can also be found from Eq. (6) (inset in Fig. 2). A line shape similar to that of Eq. (6) was obtained in the case of optical bistability owing to radiation pressure[4] but with the opposite sign of β, since the radiation pressure pushes the mirrors apart. (Note that in our experiment the effect of radiation pressure is negligible, owing to the rigid mounting of both mirrors.)

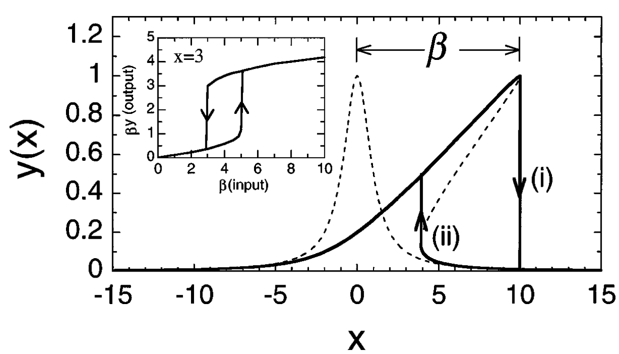

Fig. 2. Graphic solution of Eq. (6) in the form of a tilted Lorentzian ($\beta = 10$). Inset: power hysteresis curve.

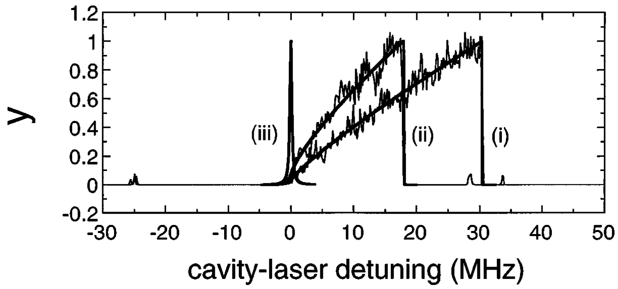

Fig. 3. Fits of cavity-transmission line shapes with the solution of Eq. (8). Experimental parameters: $P_0 = 10$ mW, $\epsilon = 0.0075$, with cavity-scan seed (i) $\dot{\omega}_c/2\pi = -0.6$ GHz/s and (ii) $\dot{\omega}_c/2\pi = -4.8$ GHz/s. The fits are obtained with $\beta = 500$ with (i) $x_0 = 3.2$ and (ii) $X_0 = 25.6$. The unmodified cavity line shape is also shown in (iii) for comparison. Small narrow blips are due to FM modulation of the probe laser at 25 MHz.

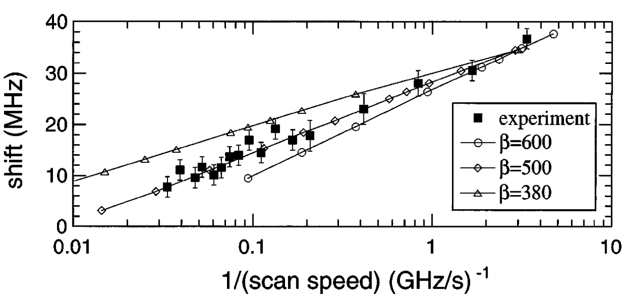

Fig. 4. Frequency shift as a function of the inverse of cavity-scan speed. The best fit is obtained with $\beta = 500$.

Suppose that the cavity is scanned nonadiabatically, i.e., that l_0 is changed more rapidly. This is the condition under which most of our experiment was performed (Fig. 3). Since it takes a certain amount of time τ for the mirror temperature to reach its equilibrium value, the thermal expansion, which depends on the temperature, is then no longer proportional to the instantaneous laser power inside the cavity. In this case the frequency shift is given by a convolution of $y(t)$ with a thermal-response function $f(t/\tau)$:

$$\frac{1}{y(x)} = 1 + \left\{x - \frac{\beta}{x_0}\int_{-\infty}^{x} y(x')f[(x-x')/x_0]\mathrm{d}x'\right\}^2, \quad (8)$$

where the response time $\tau = C_0 s w^2/\kappa$ and $x_0 = \dot{\omega}_c\tau/\gamma_c$, with $\dot{\omega}_c$ being the cavity-scan speed in radians per second. The function $f(t/\tau)$ satisfies $f(0) = 1$, $f(t/\tau) \ll 1$ for $t/\tau \gg 1$, and $\int_0^\infty f(t/\tau)\mathrm{d}t = \tau$. For $x_0 < < 1$ (adiabatic limit), Eq. (8) reduces to Eq. (6), and for $x_0 \gg 1$ (extremely rapid scan) we obtain $y(x) \rightarrow y_0(x)$.

In the experiment a probe beam is obtained from a cw Ti:sapphire laser (Coherent 899-21; $\lambda = 791$ nm), which is frequency stabilized with FM spectroscopy, as previously reported.[8] The cavity employs two supercavity mirrors ($R \simeq L \simeq 4$ mm) separated by 1.1 mm (so that $w = 30$ μm). The mirror substrate is fused silica ($\kappa = 1.4 \times 10^5$ erg/cmK, $C_{\mathrm{ex}} = 1.6 \times 10^{-6}$) with alternating layers of SiO_2 and Ta_2O_5 films deposited upon it. The total number of coating layers is 45, resulting in a coating thickness of $\sim$10 μm. The cavity finesse is measured to be 8×10^5 by a cavity-ringdown technique.[9] The length of the cavity is changed by a piezoelectric transducer upon which the mirrors are mounted [Fig. 1(a)]. The cavity was scanned over 150 MHz at various scan speeds (Fig. 3). The incident laser power was varied from 10 μW to 10 mW. We did not see any indication of laser-induced damage of the mirror coatings, even at the highest power level. The maximum temperature rise that occurs on the mirror surface is less than 1°C according to our model.

The frequency shifts measured at various scan speeds are summarized in Fig. 4. From data fits we obtain $\beta = 500 \pm 50$. Using the above values of κ and C_{ex} for fused silica, we then obtain $\mathcal{T}\mathcal{A} = 0.14 \pm 0.02$ ppm^2. From the cavity-transmission measurement, $\mathcal{T}$ is known to be 0.6 ± 0.2 ppm. We also measured the scatter directly with an integrating sphere that was specially modified for this purpose. The result is $S = 3 \pm 1$ ppm. These $\mathcal{T}$, $\mathcal{A}$, and S values are consistent with the above-mentioned finesse value.

The most remarkable feature of this technique is its high sensitivity. The frequency shift in our experiment is hundreds of times larger than the cavity linewidth. Since a frequency shift a few times larger than the cavity linewidth is easily detectable, even absorption levels much smaller than that obtained here can be measured. At a very low level of absorption, however, the sensitivity will be limited by the radiation pressure effect, which causes its own competing optical bistability. If the two mirrors are not identical, the average of two absorption coefficients will be measured. In addition, measurements of the usual input–output bistability curve and the passage and response times would be interesting from the conventional optical bistability viewpoint and are planned for future experiments.

This work is supported by the National Science Foundation under grants PHY-9512056 and CHE-9304251.

References

1. P. Meystre and M. Sargent III, *Elements of Quantum Optics* (Springer-Verlag, New York, 1991).
2. H. M. Gibbs, *Optical Bistability: Controlling Light with Light* (Academic, San Diego, Calif., 1985), and references therein.
3. A. Dorsel, J. D. McCullen, P. Meystre, E. Vignes, and H. Walther, Phys. Rev. Lett. **51,** 1550 (1983).
4. A. Gozzini, F. Maccarrone, F. Mango, I. Longo, and S. J. Barbarino, Opt. Soc. Am. B **2,** 1841 (1985).
5. P. Berman, ed., *Cavity Quantum Electrodynamics* (Academic, San Diego, Calif., 1994).
6. P. Roche, M. Commandre, L. Escoubas, J. P. Borgogno, G. Albrand, and B. Lazarides, Appl. Opt. **35,** 5059 (1996).
7. M. Commandre and P. Roche, Appl. Opt. **35,** 5021 (1996).
8. K. An, R. R. Dasari, and M. S. Feld, Appl. Phys. Lett. **66,** 2162 (1995).
9. K. An, C. Yang, R. R. Dasari, and M. S. Feld, Opt. Lett. **20,** 1068 (1995).